5A Quiz 3 : 1.6, 3.4 1.8

(1) For what values of x is the following function continuous (3 points)

$$f(x)=\frac{4}{2\sin x+1}$$

f(x) is conts on its domain, so find the domain:

Denominator can't be zero so

$2\sin x+1\neq 0 \Rightarrow \sin x\neq -1/2$

$\boxed{\text{f(x) conts for all real numbers except } x=\frac{7\pi}{6}+2\pi k,\ \frac{11\pi}{6}+2\pi k,\ k \text{ integer}}$

(2). Calculate the following limits using algebraic techniques (i.e. not a graph of table of numbers). Show work. (3 points eacj)

(c) $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x^2-4}$ = $\underline{0}$.

$=\frac{0}{-4}=0$

write lim at each step until the limit has been computed

(d) $\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}$ = $\underline{\frac{1}{10}}$.

"$\frac{0}{0}$"

$\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}\cdot\frac{\sqrt{x}+5}{\sqrt{x}+5}=\lim_{x\to 25}\frac{x-25}{(x-25)\sqrt{x}+5}$

$=\lim_{x\to 25}\frac{1}{\sqrt{x}+5}=\frac{1}{10}$

(c) $\lim_{x\to\infty}\frac{2x^2-7x-4}{x^2-16}$ = $\underline{2}$.

"$\frac{\infty}{\infty}$"

$\lim_{x\to\infty}\frac{2x^2-7x-4}{x^2-16}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}=\lim_{x\to\infty}\frac{2-\frac{7}{x}-\frac{4}{x^2}}{1-\frac{16}{x^2}}$

$=\frac{2-0-0}{1-0}$

$=2$

(d) $\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x+6}$ = ________.

Show details so it is clear you understand

$\frac{\sqrt{x^2+1}}{3x+6}=\frac{\sqrt{x^2(1+\frac{1}{x^2})}}{3x+6}=\frac{|x|\sqrt{1+\frac{1}{x^2}}}{3x+6}$

$=\frac{-x\sqrt{1+\frac{1}{x^2}}}{3x+6}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}=\frac{-\sqrt{1+\frac{1}{x^2}}}{3+\frac{6}{x}}$

$*\ (\sqrt{x^2}=|x|=-x$ since $x<0)$

you need to explain why* the minus appears.

so $\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x+6}=\lim_{x\to-\infty}\frac{-\sqrt{1+\frac{1}{x^2}}}{3+\frac{6}{x}}=\frac{-1}{3}$

(3) Given $f(x)=\begin{cases} x+2c & \text{if } x>4 \\ cx^2 & \text{if } x\le 4 \end{cases}$ find the value of c so that $\lim_{x\to 4} f(x)$ exists. Explain.

(3 points)

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} cx^2 = 16c$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} x+2c = 4+2c$$

For $\lim_{x\to 4} f(x)$ to exist, we need $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$

$$16c = 4+2c$$

$$\boxed{c = 2/7}$$

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(4) Write out the formal definition of $\lim_{x\to\infty} f(x) = -\infty$ (2 points)

If for every $M>0$ there exist an $N>0$ such that if $x>N$ then $f(x)<-M$, we say

$$\lim_{x\to\infty} f(x) = -\infty$$